

Estimating the BLA of MIMO sub-networks in simulations

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Abstract

This presentation was given on the 2015 BeNeLux Meeting on systems and Control from 24-26 march 2015 in Lommel, Belgium. The abstract for the talk can be found in the proceedings. The most important figures, needed to understand what I was saying, are extracted from the slides. At the end of each slide, I added a short summary which contains the most important messages for that slide.

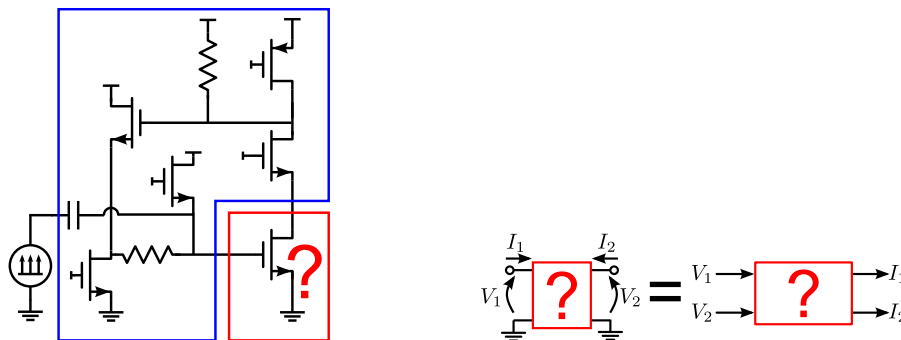
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Good afternoon everyone, I will be talking about the estimation of the Best Linear Approximation of Multiple-Input Multiple-Output sub-networks. During my presentation, I will focus on electronic circuits as examples, because it is very easy to find examples of interconnected MIMO networks there. But, as is common in system theory, the results can easily be extended towards other interconnected MIMO networks.

Electronic circuits = MIMO network...

Let me first show you how the MIMO comes into an electronic circuit. Take, as an example, a Low Noise Amplifier. It consists of several transistors and each of these transistors is non-linear. The whole circuit is excited by an input multisine which is designed to mimic the properties of the telecommunication signals the LNA will encounter in its application.

These days, one of the key specifications of electronic circuits is the amount of non-linear distortion they introduce in the signal chain. Therefore, it is of interest to study how the distortion of one of the transistors contributes to the total distortion generated by the circuit. To gain more insight into the distortion introduced by a transistor, I need to know its Best Linear Approximation or BLA.



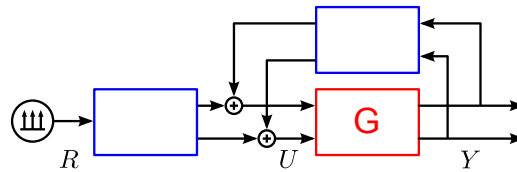
The sub-networks in an electronic circuit are Multiple-Input Multiple-Output or MIMO blocks, due to the classic port representation. For each wire going into the sub-network, we get both an input and an output. The linear behaviour of the NMOS transistor at the bottom has two wires, so it is modeled by a 2×2 MIMO Frequency Response Matrix, Its Best Linear Approximation is therefore also a MIMO BLA. If we consider the voltages at the ports as inputs and the currents flowing into the ports as outputs, the resulting frequency response functions are called Y-parameters.

It is our goal to identify this MIMO BLA of each block of interest, so we have to perform a MIMO identification where the circuit is excited by a single large signal.

- Port representation gives a MIMO BLA
- We would like to know this MIMO BLA

Excited by a single multisine

If we rephrase the problem into the circuit diagrams the control people are familiar with, we obtain the following:



We see the multisine input on the left. This multisine excites the whole system and sets the non-linear operating point. There is a Single-Input Multiple-Output non-linear system between the multisine and the input of our sub-network of interest. This block is usually referred to as generator. Around the mystery block, there's also a (non-linear) feedback network.

This general problem of a system in feedback is tackled regularly in identification. The indirect method, where we work with a known reference signal, is commonly used. First the BLA from reference to input and reference to output are estimated. Then the BLA of the system in feedback is obtained by dividing these BLAs.

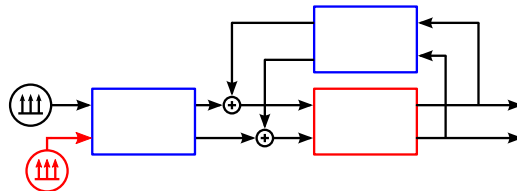
$$BLA_{\gamma} = \frac{BLA_{R \rightarrow Y}}{BLA_{R \rightarrow U}}$$

When the whole system is excited by a single multisine, we run into trouble. Because the inputs of the mystery system cannot be excited separately, the MIMO estimation is not possible.

- We obtain a MIMO system in feedback excited by a single multisine
- Classic MIMO estimation is not possible, due to the lack of excitation signals.

Classic solution: Add extra multisine

The classic solution is to add a second multisine, somewhere else in the circuit.



The non-linear operating point of the whole circuit should not be changed by this second multisine. Hence, it's power should be zero, or very small in practical applications. This is why the second multisine is referred to as a tickler multisine.

Several tricks exist to ensure that the distortion generated by the large multisine does not overwhelm the response of the system to the tickler. This tickler approach has been applied both in measurements and in simulations before, but in a carefully controlled way.

In a dynamic system, the correct tickler amplitude becomes frequency dependent. So, to be able to choose the amplitude spectrum of the tickler multisine, one needs some knowledge about the system. We are going for a push-the-button approach, so we should be looking into a method that requires no advance knowledge about the system.

- Classic approach is to add tickler multisine(s)
- The power of the tickler should be zero, not to disturb the non-linear operating point, but not too small to be above the (simulation) noise floor
- Correct choice of the tickler amplitude spectrum requires knowledge about the system.

LSSS analysis offers a solution

The solution to the possible problems introduced by the tickler can be found in linearisation. Since the amplitude of the tickler should be zero anyway, we can use linearisation to calculate its response. To understand how this will work, let us revise some of the basics of linearisation in non-linear circuits:

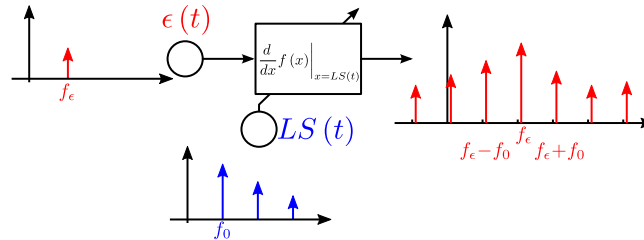
$$f(DC + \epsilon(t)) = f(DC) + \epsilon(t) \cdot \left. \frac{d}{dx} f(x) \right|_{x=DC}$$

We have a non-linear function $f(x)$ biased around a DC operating point, or quiescent point. If we want to study the way of a small signal, superimposed to the DC operating point, we apply a Taylor expansion around the operating point, to obtain the transfer function of the non-linear system.

This kind of reasoning still holds if we apply a large signal $LS(t)$ together with a small signal $\epsilon(t)$ to the system. Now the linearisation is performed around this large signal, but we still obtain the linear transfer of the small-signal through the non-linear system. This analysis is referred to as a Large-Signal Small-Signal analysis.

$$f(LS(t) + \epsilon(t)) = f(LS(t)) + \epsilon(t) \cdot \left. \frac{d}{dx} f(x) \right|_{x=LS(t)}$$

Lets use an example to clarify: The LSSS analysis is commonly used in simulations of frequency mixers, a component in a transceiver which deals with the downconversion of the high-frequency signals to baseband. In frequency mixers, a large signal Local oscillator pumps the system at a precicely controlled rate. The small-signal there is the communication signal.



The small-signal is considered to be a single sine at a frequency f_ϵ . The large signal is a periodic signal with a base frequency f_0 . The response to this small signal is a sum of sines. We get a response at f_ϵ itself, but also at $f_\epsilon \pm k f_0$. In a downconverting mixer, the transfer from f_ϵ to $f_\epsilon - f_0$ is the one of importance.

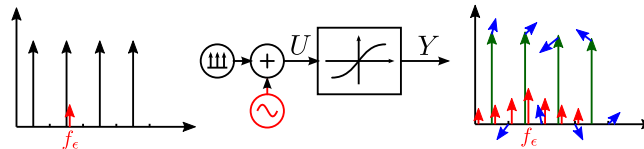
Each of these responses is linear, and is calculated by evaluating the derivative of the system in the large signal.

To conclude: a good simulator can immediately calculate the linear response of the system around a large signal operating point by calculating the derivative $\left. \frac{d}{dx} f(x) \right|_{x=LS(t)}$ without ever applying a tickler tone. This means we can calculate the response to the tickler without even tickling the circuit!

- The tickler multisine is very small, so we can look at the linearised response anyway
- LSSS analysis gives this linearised response, without danger of disturbing the non-linear operating point

The small signal sees the BLA as well

Let us look at how the Large-Signal Small-Signal analysis can be combined with the Best Linear Approximation. As an example, consider a third order non-linear system excited by an odd multisine. We also perform a Large-Signal Small-Signal analysis at a frequency f_ϵ , which is not on the frequency grid of the multisine.



We can write the output of the non-linear system at one of excited frequency bins as the sum of the linear response and the contributions from the third order Volterra kernel.

$$Y(f_k) = U(f_k) \cdot \left(\underbrace{G^{[1]}(f_k) + \sum_{i=-N}^N G^{[3]}(f_i, -f_i, f_k) \cdot |U(f_i)|^2}_{\text{Coherent}} + \underbrace{\sum_{i=-N}^N \sum_{j=-N}^N G^{[3]}(f_i, f_j, f_k) \cdot U(f_i)U(f_j)}_{\text{Non-Coherent}} \right)$$

The contributions can be split in coherent contributions, which are independent of the phase of the input signal and non-coherent contributions, which do. If the input signal has random phase, the non-coherent contributions average to zero. Then the Best Linear Approximation is given by the coherent contributions only.

Using the same Volterra tricks, we can calculate the result of the LSSS analysis. We write the input as the sum of the multisine and a single sine at frequency f_ϵ and consider the limit for the amplitude of the tickler going to zero

$$Y(f_\epsilon) = U(f_\epsilon) \cdot \left(G^{[1]}(f_\epsilon) + \sum_{i=-N}^N G^{[3]}(f_i, -f_i, f_\epsilon) \cdot |U(f_i)|^2 \right)$$

Coherent

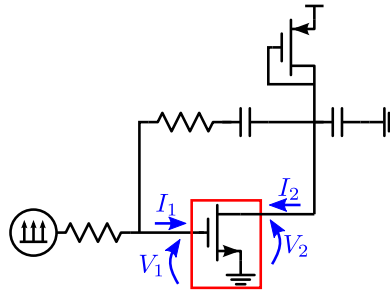
We obtain only the coherent contributions of the system, so we immediately obtain the BLA at the tickler frequency f_ϵ .

We did the same exercise up to higher orders of non-linearity and found smooth-in-the-frequency non-coherent contributions in $Y(f_\epsilon)$ at the fifth order. When the results of several LSSS analyses are averaged, these non-coherent contributions go to zero and only the BLA remains.

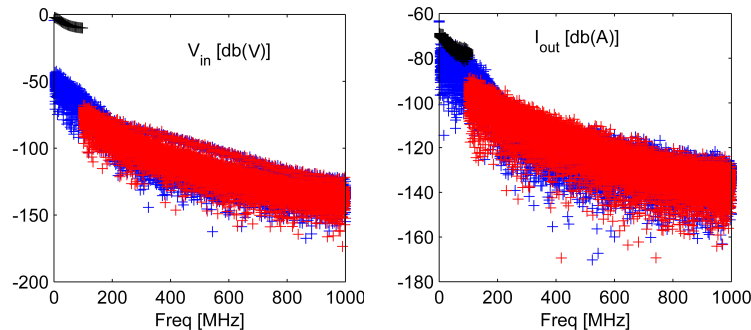
- The LSSS yields the same BLA as for the large signal multisine
- The non-coherent contributions are different at f_ϵ , and they are smooth in function of frequency

Example: CMOS amplifier

Let's apply all this theory to something practical, just to show that we didn't make an error. I use a CMOS amplifier, where I added some feedback and a heavy load capacitor to introduce significant dynamic behaviour in the system. The active load on top will introduce some non-linearity in the feedback path as well.



We drive this circuit heavily into its non-linear regime with an odd multisine voltage source at the input of the whole circuit V_{ref} . I simulate 50 different random phase realisations of the multisine using harmonic balance. Looking at the spectrum of the output current, we can clearly see that the circuit is starting to behave strongly non-linear: the signal to distortion ratio there is around XdB.

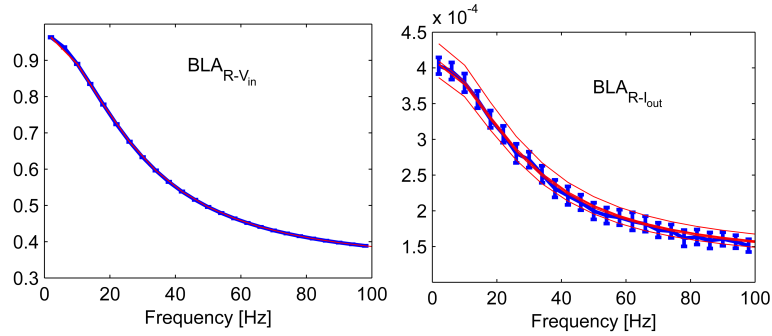


We also perform the Large-Signal Small-Signal analysis to this circuit, where the multisines are the same as for the first part of the experiment. The linearised response from V_{ref} to V_1 and I_2 is calculated at several frequencies in the frequency band of the multisine.

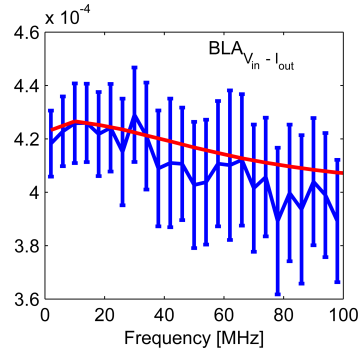
As expected, The BLA for both techniques is the same

In this Single-Input Single-Output case, we can easily compare the frequency responses obtained by both methods. We calculate the BLA from reference V_{ref} to input V_1 and from reference to output I_2 by averaging over the phase corrected signals at the input and output respectively. The amplitude of the mean and its 95% uncertainty intervals are shown in blue.

The red lines are the results for 5 different linearised responses from reference to input and output. We see they fall within the uncertainty interval set by the classic method.



The BLA of the system itself is given by the complex division of both BLAs, so there still the both methods yield the same result.

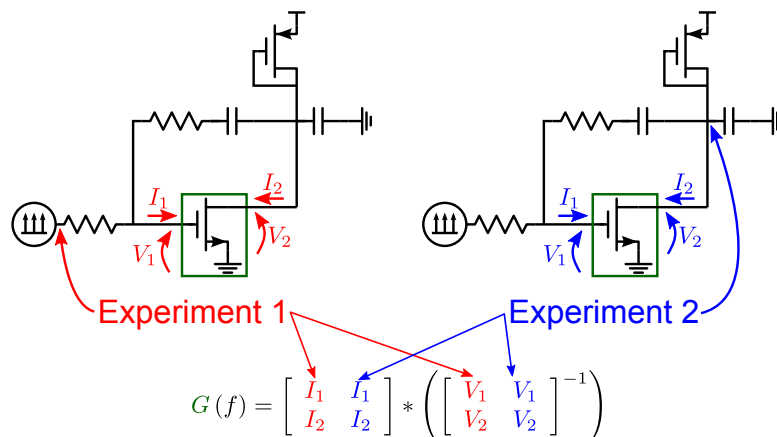


It is important to note that we used 50 phase realisations to obtain the classic BLA, while the linearised response only needed 5 realisations to obtain the same (or even lower) uncertainty.

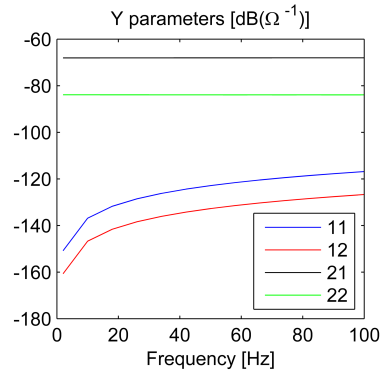
- The simulations confirm the Volterra Analysis
- Using the LSSS, we need to average over less phase realisations of the large multisine to obtain the same uncertainty

MIMO is just multiple experiments

Let's now, finally, move to our initial goal: to estimate the MIMO BLA of the sub-networks. To be able to estimate the MIMO response of a 2×2 system, we need a second experiment. To that end, we calculate the linear response from a second node in the circuit to the input and output of the system.



In our test case, I performed the second experiment from the output node. I calculate the transfer from the output node to all ports of the sub-circuit under test. Combining both Large-Signal Small-Signal analysis results in the classic MIMO input and output matrices allows to calculate the FRM by just applying an inversion.



The results look good, we clearly see that the input conductance is capacitive and that the transconductance is almost flat in function of frequency.

- Performing LSSS from a second location gives the extra experiments needed in MIMO identification
- We can estimate the MIMO BLA of the sub-circuit this way

Conclusions

Let me conclude by summarising the different points I made in this talk: The main message I'd like you guys to take home is that linearisation around a large signal is a very useful tool in the analysis of the behaviour of non-linear systems. If you have a Large-Signal Small-Signal analysis available in your favourite simulator, I recommend you start using it.

Linearisation around a multisine is strongly connected to the Best Linear Approximation. We can quickly obtain the BLA by linearising around only a few realisations of the multisine. For the MIMO BLA, the linearisation gives us a safe way to obtain the multiple experiments we need to obtain the MIMO FRM.

- Linearisation around Multisine \Rightarrow Tickler issues solved
 - No danger of disturbing the operating point
- Additional benefits:
 - Single-tone excitation \Rightarrow flexible frequency grid
 - Non-wanted contributions are smooth \Rightarrow less MS phase realisations needed
- But! you need access to Large-Signal Small-Signal analysis