$$\boxed{\ell_{\widehat{AB}} = \int_a^b \left\|\frac{d\,\vec{f}}{dt}\right\| dt \quad in\ componenten\text{: } \ell_{\widehat{AB}} = \int_a^b \sqrt{\sum_{i=1}^{n} f_i'(t)^2}\ \ dt}$$

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \Rightarrow \ \ell_{\widehat{AB}} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \quad dt$$

$$x = x(y) \Rightarrow \ \ell_{\widehat{AB}} = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} \quad dy \qquad\qquad y = y(x) \Rightarrow \ \ell_{\widehat{AB}} = \int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} \quad dx$$

$$\ell_{\widehat{AB}} = \int_a^b \sqrt{\left(\frac{d\rho}{dt}\right)^2 + \rho^{2\left(\frac{d\theta}{dt}\right)^2}} \quad dt \qquad\qquad \rho = \rho(\theta) \Rightarrow \ \ell_{\widehat{AB}} = \int_a^b \sqrt{\left(\frac{d\rho}{d\theta}\right)^2 + \rho^2} \quad d\theta$$

$$Omwentelingsopp\text{: } \ V = \pi \int_a^b f(x)^2 dx \qquad\qquad Regeloppervlak\text{: } \ V = \frac{1}{2}\int_a^b f(x)g(x)dx$$

$$\boxed{\begin{cases} x = \phi(u,v) \\ y = \ \varphi(u,v) \end{cases} \Rightarrow \iint_G \ f(x,y)dxdy = \iint_g \ f(\phi(u,v),\varphi(u,v)\left|\frac{\partial(\phi,\varphi)}{\partial(u,v)}\right| dudv \qquad \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}}$$

$$poolcoordinaten\text{: } j = \rho \qquad\qquad bolcoordinaten = j = \ \rho^2 sin\theta$$

$$\boxed{\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases} \Rightarrow \vec{r} = \ \vec{r}(u,v) \Rightarrow \ Opp(S) = \iint_R \ \left\|\frac{\partial\vec{r}}{\partial u} \times \frac{\partial\vec{r}}{\partial v}\right\| dudv = \iint_g \ \sqrt{EG - F^2} dudv}$$

$$E = \ \frac{\partial\vec{r}}{\partial u}.\frac{\partial\vec{r}}{\partial u} \qquad F = \ \frac{\partial\vec{r}}{\partial u}.\frac{\partial\vec{r}}{\partial v} \qquad G = \ \frac{\partial\vec{r}}{\partial v}.\frac{\partial\vec{r}}{\partial v}$$

$$GREEN - RIEMANN\text{: } \oint_{C^+} P(x,y)dx + Q(x,y)dy = \iint_G \ \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dO$$

$$STOKES\text{: } \iint_S \ (rot\ \vec{v}.\vec{n})dO = \oint_C \ \vec{v}(\vec{r})d\,\vec{r}$$

$$OSTROGRADSKY\text{: } \iint_S \ (\vec{v}.\vec{n})\ dO = \iiint_G \ div\ \vec{v}\ dv$$

Positieve reeksen

$\underline{a)\ Vergelijkingskenmerken:}$

$$\exists k \in \mathbb{R}: \forall n > N: u_n \leq k v_n \ \ dan \sum u_n \ convergent \Rightarrow \sum v_n \ convergent$$

$$als \lim_{n\to\infty} \frac{u_n}{v_n} = k \ met \ k \in \mathbb{R}^+ \ dan \sum u_n \ en \sum v_n \ samen \ conv. of \ div.$$

$$als \lim_{n\to\infty} \frac{u_n}{v_n} = 0 \ dan \sum v_n \ conv. \ \Rightarrow \ \sum u_n$$

$$als \lim_{n\to\infty} \frac{u_n}{v_n} = +\infty \ dan \sum u_n \ conv. \ \Rightarrow \ \sum v_n$$

$$referentiereeksen:$$

$$Harmonische\ reeksen: \sum \frac{1}{n^\alpha} \quad \begin{cases} \alpha > 1 \ \Rightarrow \ convergent \\ \alpha \leq 1 \ \Rightarrow \ divergent \end{cases}$$

$$Meetkundige\ reeksen: \sum a^n \begin{cases} a < 1 \ \Rightarrow \ convergent \\ a > 1 \ \Rightarrow \ divergent \end{cases}$$

$$Reeksen\ van\ Bertrand: \sum \frac{1}{n(\ln n)^\beta} \begin{cases} \beta > 1 \Rightarrow convergent \\ \beta \leq 1 \Rightarrow \ divergent \end{cases}$$

$\underline{b)\ Integraalkenmerk:}$

$$Stel\ f: [a, +\infty[\ \rightarrow \ \mathbb{R}^+ \ continu\ en\ niet\ stijgend, dan \int_a^{+\infty} f(t)dt \ \ conv. \ \Leftrightarrow \ \sum f(n) \ \ conv.$$

$\underline{c)\ Wortelkenmerk\ van\ Cauchy}$

$$\lim_{n\to\infty} \sqrt[n]{u_n} \quad \begin{cases} < 1 \ \Rightarrow \sum u_n \ convergent \\ > 1 \Rightarrow \sum u_n \ \ divergent \\ = 1 \ \Rightarrow ?? \end{cases}$$

$\underline{d)\ Convergentiekenmerk\ van\ d'Alembert}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} \quad \begin{cases} < 1 \ \Rightarrow \sum u_n \ convergent \\ > 1 \Rightarrow \sum u_n \ \ divergent \\ = 1 \ \Rightarrow ?? \end{cases}$$

Willekeurige reeksen

$a)$ *Eigenschap en definitie:*

$$\sum |u_n| \ conv. \ \Rightarrow \sum u_n \ conv.$$

$$Als \sum |u_n| \ conv. \ dan\ noemen\ we \sum u_n \ absoluut\ conv.$$

$$Als \sum u_n \ convergent\ is\ maar\ niet\ absoluut\ conv. dan\ is \sum u_n \ relatief\ conv.$$

$b)$ *Kenmerk van Leibniz (alternerende reeksen)*

$$Als \lim_{n \to \infty} u_n = 0\ en\ \exists\ N: \forall n > N:\ \ u_{n+1} \ \leq \ u_n (u_n niet\ stijgend)\ \ dan\ is \sum (-1)^n u_n \ conv.$$

Reeksen van Functies

Criterium van Weierstrass:

$$Als\ |u_n(x)| \ \leq \ m_n \ \ \forall n \ > N\ en\ x \in A\ en\ als \sum m_n \ conv.\ numerieke\ reeks\ is$$

$$dan\ is \sum u_n(x) \ \ uniform\ convergent$$

Machtreeksen

$$Definitie: \sum a_n (x-a)^n \ \ met\ a_n, a \ \in \mathbb{R} \ machtreeks.$$

$$Convergentiestraal\ R: \ \frac{1}{R} = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \sqrt[n]{|a_n|}$$

$$\sum a_n x^n \ \ is \begin{cases} convergent \ \ als\ |x| < R \\ divergent \ \ \ als\ |x| > R \\ \ \ ?? \ \ \ \ \ \ \ \ als\ x \ \in \{-R, R\} \end{cases}$$

Juiste differentiaalvergelijkingen

$$(x,y)dx + Q(x,y)dy = 0 \; als \; \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \; \Rightarrow \; opl: \int_{x_0}^{x} P(t, y_0)dt + \int_{y_0}^{y} Q(x,u)du = \; c^{te}$$

Differentiaalvergelijking met gescheiden veranderlijken

$$\frac{dy}{dx} = f(x,y) \Leftrightarrow \; P(x)dx = Q(y)dy \; \Rightarrow \; \int P(x)dx = \; \int Q(y)dy$$

Homogene differentiaalvergelijking

$$P(x,y)dx + Q(x,y)dy = 0 \; met \; P(x,y) en Q(x,y) homogene \; veeltermen \; van \; graad \; m$$

$$y = zx \; \Rightarrow \; dy = xdz + zdx \; substitueren \; en \; dan \; delen \; door \; x^m \; \Rightarrow \; gescheiden \; veranderlijkn$$

Herleiden tot homogene differentiaalvergelijking

$$\frac{dy}{dx} = f\left(\frac{ax + by + c}{a'x + b'y + c'}\right) \qquad stel \quad \ell = ax + by + c \quad en \quad \ell' = a'x + b'y + c'$$

$\underline{\ell \parallel \ell'}$

$$\frac{a'}{a} = \frac{b'}{b} = \lambda \; \Rightarrow \; \frac{dy}{dx} = f\left(\frac{ax + by + c}{\lambda(ax + by) + c'}\right)$$

$$stel \;\; u = ax + by \;\; \Rightarrow homogeen$$

$\underline{\ell \nparallel \ell'}$

$$\ell \; en \; \ell' snijden \; in \; (x_0, y_0)$$

$$stel \begin{cases} x = t + x_0 \\ y = u + y_0 \end{cases} \Rightarrow \; homogeen$$

Lineaire differentiaalvergelijkingen van 1ste orde

$$a(x)\frac{dy}{dx} + b(x)y = d(x)$$

1) $Homogeen: \quad a(x)\frac{dy}{dx} + \; b(x)y = 0 \; \Rightarrow \; y_h = cf(x)$
2) $Particulier: \;\; y_p = c(x)f(x) \; \Rightarrow \;\; a(x)c'(x)f(x) = d(x) \Rightarrow \; c(x) = \int \frac{d(x)}{a(x)f(x)} dx$

Bernoulli

$$a(x)\frac{dy}{dx} + b(x)y = d(x)y^m \quad stel \; z = y^{1-m} \;\; \Rightarrow \; z' = (1-m)y^{-m}y'$$

$$\Rightarrow \; a(x)\frac{z'}{1-m} + b(x)z = d(x) \quad lineair$$

Differentiaalvergelijking van eerste orde met willekeurige graad

Ontbinden

$$F(x,y,y') = \big(y' - f_1(x,y)\big)\big(y' - f_2(x,y)\big) \ldots \big(y' - f_n(x,y)\big) = 0 \;\Rightarrow\; y' = f_i(x,y)$$

$$algemene\ oplossing\ is\ \prod_{i=1}^{n} G_i(x,y,c_i) \ = 0$$

Overgang naar de parametervorm

1) $y\ ontbreekt: F(x,y') = 0\ \begin{cases} y' = \psi(t) \\ x = \phi(t) \end{cases} \Rightarrow\ y = \int \psi(t)\phi'(t)dt$
2) $x\ ontbreekt: F(y,y') = 0\ \begin{cases} y' = \psi(t) \\ y = \phi(t) \end{cases} \Rightarrow\ x = \int \frac{\phi'(t)}{\psi(t)} dt$
3) $homogeen\ in\ x\ en\ y: F\left({}^{y}/_{x}\ , y'\right) = 0\ \begin{cases} y' = \psi(t) \\ {}^{y}/_{x} = \phi(t) \end{cases} \Rightarrow\ x = c * \exp \int \frac{\phi'(t)}{\psi(t) - \phi(t)} dt$

Afleiding en eliminatie

$$F(x,y,y') = 0 \;\Rightarrow\; y = \phi(x,y')\ \ stel\ y' = p\ \;\Rightarrow\; y = \phi(x,p) \;\Rightarrow\; y' = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial p}p' = p$$

$$dit\ is\ een\ vgl\ van\ eerste\ orde\ en\ eerste\ graad\ met\ onbekende\ p.$$

Vergelijking van Lagrange

$$\boxed{y = x\phi(y') + \psi(y')}\ \ stel\ y' = p \;\Rightarrow\; y = x\phi(p) + \psi(p)\ \;\Rightarrow\; p = \phi(p) +\ \big(x\phi'^{(p)} + \psi'^{(p)}\big)p'$$

$$\Rightarrow\ \big(p -\ \phi(p)\big)\frac{dx}{dp} - \phi'(p)x = \psi'(p)\ dit\ is\ een\ lineare\ diffvgl\ met\ onbekende\ x$$

Vergelijking van Clairaut

$$\boxed{y = xy' + \psi(y')}\ is\ een\ vgl\ van\ Lagrange\ met\ \phi(p) = p. we\ vinden\ dus\ \big(x + \psi'(p)\big)\frac{dp}{dx} = 0$$

1) $integreren\ van\ \frac{dp}{dx} = 0\ geeft\ p = c\ en\ de\ algemene\ integraal\ is\ y = cx + \psi(c)$
2) $de\ vglg\ x + \psi'(p)\ geeft\ een\ singuliere\ opl\ in\ parametervorm\ \begin{cases} x = -\psi'(p) \\ y = px + \psi(p) \end{cases}$

Vergelijking van Euler

$$(ax+b)^n y^{(n)} + a_1(ax+b)^{n-1}y^{(n-1)} + \cdots + a_{n-1}(ax+b)y' + a_0 y = f(x)$$

$$substitutie: u = \ln|ax+b|\ \ geeft\ een\ diffvgl\ met\ constante\ coefficienten.$$